

Nucleon propagation through nuclear matter in chiral effective field theory

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Abstract. We treat the propagation of a nucleon in nuclear matter by evaluating the ensemble average of the two-point function of the nucleon currents in the framework of chiral effective field theory. We first derive the effective parameters of the nucleon to one loop. The resulting formula for the effective mass has been known since before and gives an absurd value at normal nuclear density. We then modify it following Weinberg's method for the two-nucleon system in the effective theory. Our results for the effective mass and the width of the nucleon are compared with those in the literature.

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1 Introduction

Chiral perturbation theory is the effective theory of the strong interactions, based only on the approximate chiral symmetry of the QCD Lagrangian and on the assumption of spontaneous breaking. While this theory is eminently successful in dealing with pionic processes [1–3], and also, to a great extent, with those of pions and a single nucleon [4], its direct application to processes involving two or more nucleons is problematic (for a recent review, see [5]; see also an earlier review in [6]). As first pointed out by Weinberg [7, 8], the contribution of the graphs for such processes grow with increasing number of loops, in gross violation to the power counting rule. This failure of power counting is, of course, only to be expected, as precisely the sum over such (an infinite number of) loop graphs would give rise to bound states such as the deuteron and the virtual bound state in the two-nucleon system.

A better analysis of the graphs can be made in the old fashioned perturbation theory [7, 8]. Here it is the loop diagrams containing one or more pure-nucleon intermediate states that lead to a divergence of the perturbation expansion, while those with intermediate states containing at least one pion obey the power counting rule. Accordingly Weinberg [7, 8] proposes to construct first the effective potential from those connected graphs of the T -matrix that do not contain any pure-nucleon intermediate state. Then the graphs with pure-nucleon intermediate states are summed over by the Lippmann–Schwinger integral equation for the T -matrix with this effective potential.

In this work we apply this procedure to find the effective mass of the nucleon in nuclear matter. The one-loop formula has long been known [9], giving the mass-shift as the nucleon number density times a constant, depending on the parameters of the effective Lagrangian to leading order. When expressed in terms of NN scattering lengths, this constant, and hence the mass-shift, becomes too large to be acceptable.

What goes wrong with the one-loop result is clearly the approximation of the appropriate NN scattering amplitude that is present in the formula by merely the s -wave scattering lengths. To improve this amplitude within the effective theory, the Weinberg analysis suggests that we regard the constant NN amplitude as the effective potential to leading order and solve the Lippmann–Schwinger equation for the complete amplitude. Replacing the constant amplitude in the one-loop formula by this solution, we get a greatly reduced value for the effective nucleon mass. We also find the effect of including phenomenologically the effective ranges in the formula [10, 11].

We first derive in some detail the old result for the effective mass of the nucleon, using the real time formulation of field theory in matter (for a review, see [12]). We begin with the ensemble average of the two-point function of the nucleon currents [13, 14]. It is evaluated to one loop with vertices from the effective Lagrangian [7, 8], using the method of external fields [2, 3]. The pole term of the nucleon immediately gives the effective mass to leading order in the usual power counting. (We also get the residue at the pole, but it contains an unknown coupling constant.) The constant scattering amplitude contained in it is then improved upon as described above.

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In Sect. 2 we obtain the pieces of the effective Lagrangian that will be needed in evaluating the one-loop graphs. We use the Dirac, rather than the Pauli, spinor for the nucleon. The advantage of this relativistic treatment is a clear separation of the vacuum and the density dependent parts of the nucleon propagator, the latter part containing the on-shell delta-function. In Sect. 3 we evaluate the graphs to obtain the leading terms for the effective parameters of the nucleon in nuclear matter. We modify the mass-shift formula in Sect. 4. Finally, Sect. 5 includes some comments on our work. In Appendix A we describe the dispersion integral method to obtain the density dependent contributions from the one-loop graphs. Appendix B gives the result of the partial wave expansion of the spin averaged NN scattering amplitude in the forward direction.

2 Chiral Lagrangian

We study the propagation of the nucleon in nuclear matter by analysing the two-point function of the nucleon current $\eta(x)$ [13, 14], built out of three quark fields so as to have the quantum numbers of the nucleon. In this work we do not need to spell out the form of this current; it suffices only to note its transformation under the chiral symmetry group $SU(2)_R \times SU(2)_L$,

$$\eta_R \rightarrow g_R \eta_R, \quad \eta_L \rightarrow g_L \eta_L, \quad g_R \in SU(2)_R, \quad g_L \in SU(2)_L, \quad (1)$$

where the subscripts R and L on η denote its right- and the left-handed components, $\eta_{R,L} = \frac{1}{2}(1 \pm \gamma_5)\eta$.

To get the low energy structure of vertices with the nucleon current in the effective theory, we couple it to an external (spinor) field $f(x)$, thereby extending the original QCD Lagrangian $\mathcal{L}_{\text{QCD}}^{(0)}$ to

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{QCD}}^{(0)} + \bar{f}\eta + \bar{\eta}f. \quad (2)$$

Writing in terms of R and L components,

$$\bar{f}\eta = \bar{f}_R \eta_L + \bar{f}_L \eta_R,$$

we see that such a term is chirally invariant if the external fields $f(x)_{R,L}$ transform oppositely to $\eta(x)_{R,L}$,

$$f_R \rightarrow g_L f_R, \quad f_L \rightarrow g_R f_L. \quad (3)$$

The transformation rules for the pion and the nucleon fields are known [2, 3, 15, 16]. One can define three different quantities involving the pion triplet, namely U , u ($u^2 = U$) and $u_\mu = iu^\dagger(\partial_\mu u)u^\dagger$, transforming as

$$\begin{aligned} U &\rightarrow g_R U g_L^\dagger, \\ u &\rightarrow g_R u h^\dagger = h u g_L^\dagger, \\ u_\mu &\rightarrow h u_\mu h^\dagger, \end{aligned} \quad (4)$$

where h is an element of the unbroken subgroup, $h \in SU(2)_V$. The nucleon field $\psi(x)$ transforms according to

the isospin $\frac{1}{2}$ representation of $SU(2)_V$,

$$\psi \rightarrow h\psi.$$

We can now construct the effective Lagrangian for the nucleon–nucleon system including pions in the presence of the external field f . Its terms may be put into three groups,

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\psi} + \mathcal{L}_{\psi^4} + \mathcal{L}_f, \quad (5)$$

which we now write down one by one. $\mathcal{L}_{\pi\psi}$ is given by the familiar terms,

$$\mathcal{L}_{\pi\psi} = \frac{F_\pi^2}{4} \partial_\mu U \partial^\mu U^\dagger + \bar{\psi}(i\partial - m)\psi + \frac{1}{2} g_A \bar{\psi} \psi \gamma_5 \psi. \quad (6)$$

Here g_A is the constant ($g_A = 1.26$) appearing in the neutron beta decay. We choose the explicit representation $U = \exp(i\phi^a \tau^a / F_\pi)$, where ϕ^a are the hermitian pion fields ($a = 1, 2, 3$), τ^a , the Pauli matrices and F_π , the pion decay constant ($F_\pi = 93 \text{ MeV}$). In the following we do not need vertices with pion fields only.

The piece \mathcal{L}_{ψ^4} , giving the leading quartic interaction of nucleons, has been written by Weinberg as [7, 8],

$$\mathcal{L}_{\psi^4} = -\frac{C_S}{2} (N^\dagger N)^2 - \frac{C_T}{2} (N^\dagger \boldsymbol{\sigma} N)^2 + \dots,$$

where C_S and C_T are constants, and $N(x)$ is the Pauli spinor (and isospinor) for the nucleon and ‘...’ refers to terms with two and more derivatives. As we intend to work with the Dirac spinor $\psi(x)$, we rewrite it as

$$\mathcal{L}_{\psi^4} = -\frac{C_S}{8} \{\bar{\psi}(1 + \gamma_0)\psi\}^2 - \frac{C_T}{8} \{\bar{\psi}(1 + \gamma_0)\boldsymbol{\gamma}\gamma_5\psi\}^2 + \dots \quad (7)$$

In the piece \mathcal{L}_f , dependent on the external field, we need two couplings, namely, ones that are linear and cubic in ψ ,

$$\mathcal{L}_f = \mathcal{L}_{f\psi} + \mathcal{L}_{f\psi^3}.$$

Using the above transformation rules, the piece linear in ψ may be written as

$$\begin{aligned} \mathcal{L}_{f\psi} &= \lambda(\overline{u f_R})\psi + \lambda'(\overline{u^\dagger f_L})\psi + \text{h.c.} \\ &= \frac{\lambda}{2} \bar{f}(1 - \gamma_5)u^\dagger\psi + \frac{\lambda'}{2} \bar{f}(1 + \gamma_5)u\psi + \text{h.c.}, \end{aligned} \quad (8)$$

where invariance under parity requires $\lambda = \lambda'$. Following the construction of \mathcal{L}_{ψ^4} , we write the other piece as

$$\begin{aligned} \mathcal{L}_{f\psi^3} &= \frac{A_S}{4} \bar{\psi}(1 + \gamma_0)\psi \bar{f}(1 + \gamma_0)\psi \\ &\quad + \frac{A_T}{4} \bar{\psi}(1 + \gamma_0)\boldsymbol{\gamma}\gamma_5\psi \bar{f}(1 + \gamma_0)\boldsymbol{\gamma}\gamma_5\psi + \text{h.c.} \end{aligned} \quad (9)$$

Two other possible non-derivative terms involving $\boldsymbol{\tau}$ do not appear here for the same reason as they did not in (6), namely that the anticommutativity of the nucleon field allows each of these terms to be written as linear combinations of the two terms written above. Clearly here also there are terms with two or more derivatives on the nucleon field.

3 One-loop formula

We start with the ensemble average of the two-point function of the nucleon currents in symmetric nuclear matter,

$$\Pi(q) = i \int d^4x e^{iqx} \langle T \eta(x) \bar{\eta}(0) \rangle, \quad (10)$$

where for any operator O , $\langle O \rangle = \text{Tr}[\rho O] / \text{Tr} \rho$, $\rho = e^{-\beta(H - \mu N)}$, H being the Hamiltonian of the system, β the inverse temperature, and N the nucleon number operator with chemical potential μ . In general, we deal here with functions of two variables, namely $q_0 \equiv E$ and $|\mathbf{q}|$.

In the real time version of quantum field theory in a medium, the two-point function assumes the form of a 2×2 matrix. But the dynamics is given essentially by a single analytic function, which is determined by the 11-component itself. Denoting the 11-component of the matrix amplitude by $F_{11}(E, \mathbf{q})$, the corresponding analytic function $F(E, \mathbf{q})$ has the spectral representation in E at fixed \mathbf{q} (for a review, see [12]),

$$F(E, \mathbf{q}) = \frac{1}{\pi} \int \frac{\coth(\beta(E' - \mu)/2) \text{Im} F_{11}(E', \mathbf{q})}{E' - E - iE'\epsilon} dE', \quad (11)$$

where the integral runs over the cuts of $F_{11}(E, \mathbf{q})$.

The simplest example of a two-point function in a nuclear medium is, of course, the free nucleon propagator. Its 11-component is given by [17]¹

$$\begin{aligned} \frac{1}{i} S(p)_{11} &\equiv \int d^4x e^{ipx} \langle T \psi(x) \bar{\psi}(0) \rangle_{11} \\ &= (\not{p} + m) \left[\frac{i}{p^2 - m^2 + i\epsilon} \right. \\ &\quad \left. - \{ n^-(\omega_p) \theta(p_0) + n^+(\omega_p) \theta(-p_0) \} \right. \\ &\quad \left. \times 2\pi \delta(p^2 - m^2) \right], \end{aligned} \quad (12)$$

where the n^\mp are the distribution functions for the nucleons and antinucleons, $n^\mp(\omega_p) = \{e^{\beta(\omega_p \mp \mu)} + 1\}^{-1}$, with $\omega_p = \sqrt{m^2 + p^2}$. The corresponding analytic function $S(p)$, as given by (11), is

$$S(p) = -(\not{p} + m) \frac{1}{p^2 - m^2 + i\epsilon},$$

which is independent of n^\mp and identical to the free propagator in vacuum.

In the following we shall work in the limit of zero temperature, when the distribution functions become $n^- \rightarrow$

$\theta(\mu - \omega_p)$ and $n^+ \rightarrow 0$. Then the nucleon number density, \bar{n} is given by

$$\bar{n} = 4 \int \frac{d^3p}{(2\pi)^3} \theta(\mu - \omega_p) = \frac{2p_F^3}{3\pi^2},$$

where p_F is the Fermi momentum, related to the chemical potential μ by $\mu^2 = m^2 + p_F^2$. We want to calculate the density dependent part of $\Pi(q)$ in the low energy region to first order in \bar{n} . To this end, we draw all the Feynman graphs with one loop containing a nucleon line. In addition to the single nucleon line forming a loop, we include also the loop containing an additional pion line to account for the singularities with the lowest threshold. They are depicted in Fig. 1 along with the free propagator of graph (a).

Before we work out the graphs, it is possible to simplify the piece $\mathcal{L}_{f\psi^3}$ in the Lagrangian, needed for the vertices in graphs (c) and (d) of Fig. 1. As two of the nucleon fields are contracted at the vertex itself, we can do so already in $\mathcal{L}_{f\psi^3}$ and retain only the density dependent part in the contraction. Thus, referring to (2), we get from (8) and (9) the complete expression for η to leading order in the effective theory as follows:

$$\begin{aligned} \eta &= \lambda \left\{ 1 + \frac{i\pi \cdot \tau}{2F_\pi} \gamma_5 + \zeta \bar{n} (1 + \gamma_0) \right\} \psi, \\ \zeta &= \frac{3}{8\lambda} (A_S - A_T). \end{aligned} \quad (13)$$

We shall work at $\mathbf{q} = 0$, when the free propagation graph (a) takes the form

$$\Pi(E)_{(a)} = \lambda^2 S(E) = -\frac{\lambda^2}{E - m + i\epsilon} \frac{1}{2} (1 + \gamma_0), \quad (14)$$

in the vicinity of the nucleon pole. We now calculate the corrections to it produced by the rest of the graphs.

To evaluate the self-energy diagram (b), we rewrite the four-nucleon interaction term given by (6) conveniently as

$$\mathcal{L}_{\psi^4} = -\frac{1}{8} \sum_{i=S,T} C_i \Gamma_{AB}^i \Gamma_{CD}^i (\bar{\psi}_A \psi_B) (\bar{\psi}_C \psi_D), \quad (15)$$

where $\Gamma^S = (1 + \gamma_0)$ and $\Gamma^T = (1 + \gamma_0) \gamma \gamma_5$. Then the two-point function for this diagram is given by

$$\Pi(E)_{(b)} = -\lambda^2 S(E) \sigma S(E). \quad (16)$$

Here the self-energy σ is a constant given by

$$\begin{aligned} \sigma &= -\frac{i}{4} \sum_{i=S,T} C_i \int \frac{d^4p}{(2\pi)^4} \\ &\quad \times \{ -2 \text{tr}(S(p)_{11} \Gamma^i) \Gamma^i + \Gamma^i S(p)_{11} \Gamma^i \}, \end{aligned} \quad (17)$$

where the $\text{tr}(\text{ace})$ is over the γ -matrices. On inserting the density dependent part of S_{11} from (12), σ can immediately be evaluated to give

$$\Pi(E)_{(b)} = -\frac{3\lambda^2}{4} (C_S - C_T) \bar{n} \frac{1}{(E - m)^2} \frac{1}{2} (1 + \gamma_0). \quad (18)$$

¹ Going over to the non-relativistic limit and omitting an antiparticle contribution, it reduces to the more familiar form [18]

$$-S_{11}(p) = \frac{\theta(p - p_F)}{p_0 - \omega_p + i\epsilon} + \frac{\theta(p_F - p)}{p_0 - \omega_p - i\epsilon},$$

where the vacuum and the medium contributions are mixed up.

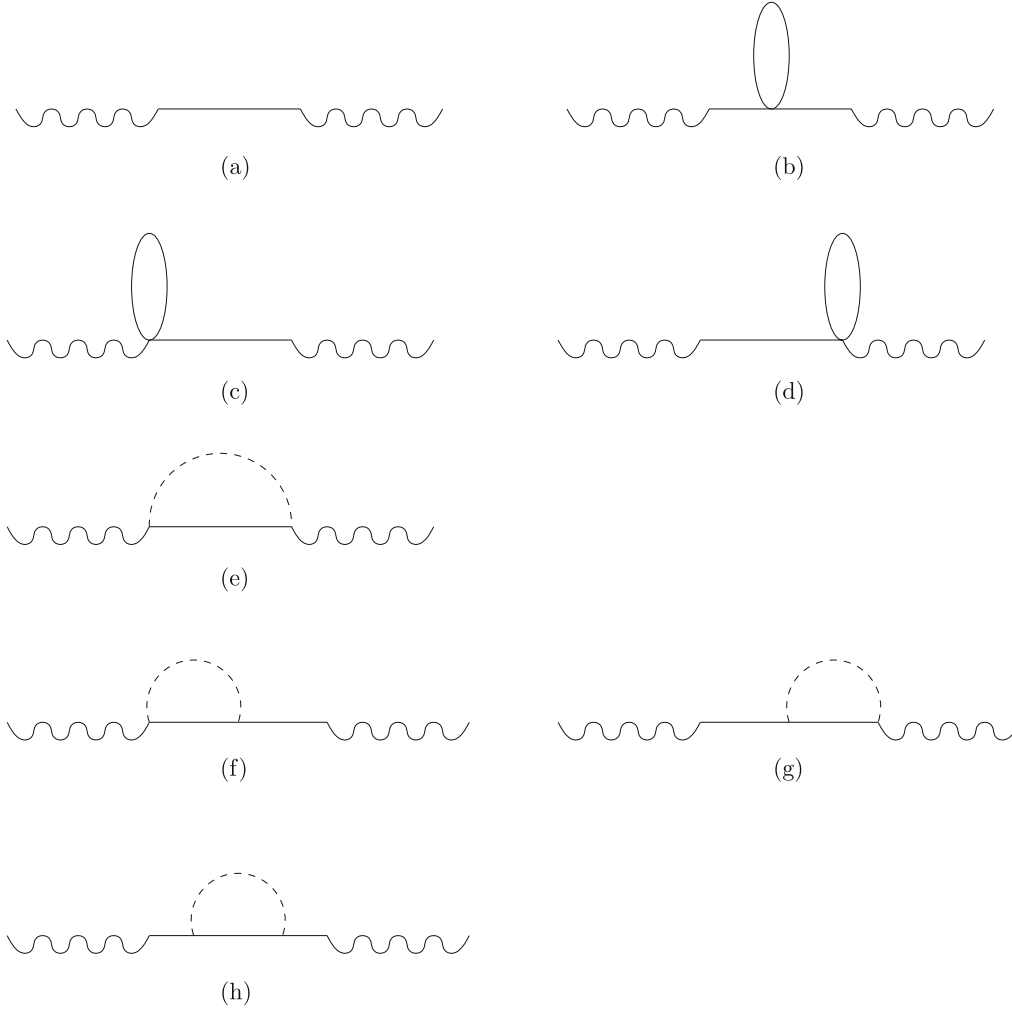


Fig. 1. Free propagator and one-loop Feynman graphs for the two-point function in the low energy region. The *wavy line* represents the nucleon current, while the *continuous* and the *dashed internal lines* are for nucleon and pion propagation

Note the absence of a simple pole in this contribution. From the constant vertex graphs (c) and (d), we get

$$\Pi(E)_{(c)+(d)} = -4\lambda^2 \zeta \bar{n} \frac{1}{E-m} \frac{1}{2} (1 + \gamma_0). \quad (19)$$

Graph (e) has the two-particle (π and N) intermediate state. It is evaluated in detail in Appendix A. As expected, it does not give rise to a pole at $E = m$. Following similar steps, we get the contributions of the remaining graphs as follows:

$$\Pi(E)_{(f)+(g)} = \frac{3\lambda^2 g_A \bar{n}}{16mF_\pi^2} \frac{1}{E-m} \frac{1}{2} (1 + \gamma_0) + \dots, \quad (20)$$

$$\Pi(E)_{(h)} = -\frac{3\lambda^2 g_A^2 \bar{n}}{16F_\pi^2} \frac{1}{(E-m)^2} \frac{1}{2} (1 + \gamma_0) + \dots, \quad (21)$$

where the ellipsis denotes non-pole terms. Again observe that graph (h) does not give rise to a simple pole.

Collecting the results for the simple and the double poles at $E = m$, we find the vacuum pole (14) to be modified in a nuclear medium to

$$-\frac{\lambda^{*2}}{E - m^* + i\epsilon} \frac{1}{2} (1 + \gamma_0),$$

where

$$\lambda^* = \lambda \left\{ 1 - \left(\frac{3g_A}{8mF_\pi^2} - 2\zeta \right) \bar{n} \right\}, \quad (22)$$

$$m^* = m + \frac{3}{4} \left(C_S - C_T + \frac{g_A^2}{4F_\pi^2} \right) \bar{n}. \quad (23)$$

The constant $(C_S - C_T)$ can directly be related to experiment, but no such relation appears to exist for ζ . As is known already [9], and as we shall also see below, the formula for m^* is unacceptable at normal nuclear density.

4 Modified formula

If we note that the nucleon mass-shift formula (23) is given by graphs (b) and (h) of Fig. 1 and recall the presence of the mass-shell delta-function in the density dependent part of the nucleon propagator (12), it is easy to guess that the coefficient of \bar{n} in this formula must be related to some on-shell NN scattering amplitude at threshold. To obtain the actual relation, we evaluate the scattering amplitude in the same chiral Lagrangian framework as that in Sect. 2. In the

tree approximation the contributing Feynman graphs are shown in Fig. 2.

We choose to calculate the spin averaged amplitude in the forward direction,

$$\begin{aligned}\overline{M}(p_1, p_2 \rightarrow p_1, p_2) \\ = \frac{1}{4} \sum_{\sigma_1, \sigma_2} M(p_1, \sigma_1; p_2, \sigma_2 \rightarrow p_1, \sigma_1; p_2, \sigma_2).\end{aligned}$$

The propagating nucleon, say, a proton, may scatter with a proton or a neutron in the medium. Thus, the isospin structure of \overline{M} in a symmetric medium must be given by

$$\overline{M} = \overline{M}_{pp \rightarrow pp} + \overline{M}_{pn \rightarrow pn}.$$

With this spin and isospin structure, the amplitude \overline{M} as calculated from the graphs of Fig. 2 is ($Q = p_2 - p_1$),

$$\begin{aligned}\overline{M} &= \frac{1}{2} \sum_{i=S,T} C_i \{ -\text{tr } \Gamma_i(\not{p}_1 + m) \Gamma_i(\not{p}_2 + m) \\ &\quad + 2\text{tr } \Gamma_i(\not{p}_1 + m) \cdot \text{tr } \Gamma_i(\not{p}_2 + m) \} \\ &\quad + \frac{3}{4} \left(\frac{g_A}{2F_\pi} \right)^2 \text{tr}(\not{p}_1 + m) \not{Q} \gamma_5 \\ &\quad \times (\not{p}_2 + m) \not{Q} \gamma_5 / (Q^2 - m_\pi^2) \\ &= -6m^2 \left\{ (C_S - C_T) \frac{(E_1 + m)(E_2 + m)}{4m^2} + \frac{g_A^2}{4F_\pi^2} \right\},\end{aligned}\quad (24)$$

E_1 and E_2 being the energies of the two nucleons. (Graph (b) in Fig. 2 does not contribute to \overline{M} .) Thus, at threshold \overline{M} involves the same combination of constants as those in (23) for m^* .

As shown in Appendix B, the amplitude \overline{M} at threshold can be expressed in terms of the s -wave spin-singlet and -triplet scattering lengths a_1 and a_3 , so one gets

$$m^* - m = \frac{3\pi}{2m} (a_1 + a_3) \bar{n}. \quad (25)$$

With their experimental values as quoted in the appendix and at normal nuclear density ($p_F = 1.36 \text{ fm}^{-1}$), it gives $m^* - m = -620 \text{ MeV}$ [9], which is unacceptably large.

To look for the source of the problem, it is suggestive to rewrite (23) and (25) as

$$\begin{aligned}m^* - m &= -\frac{1}{m} \int_0^{p_F} \frac{d^3p}{(2\pi)^3 2m} \left\{ -6m^2 \left(C_S - C_T + \frac{g_A^2}{4F_\pi^2} \right) \right\} \\ &= -\frac{1}{m} \int_0^{p_F} \frac{d^3p}{(2\pi)^3 2m} \{ -12\pi m (a_1 + a_3) \}.\end{aligned}\quad (26)$$

At this point we recall Weinberg's method for the two-nucleon amplitude. The constants in the curly brackets in the integrands above represent the scattering amplitude in terms of the s -wave scattering lengths. Clearly this approximation is too bad in the low energy region, as it does not reproduce the nearby poles due to the bound and virtual states. But we can treat these constants as effective potentials (to lowest order) and solve the Lippmann-Schwinger equation with it to get a better representation of the partial wave amplitudes. As Weinberg himself shows [7, 8], the solution in this case turns out to be the unitarised version of the potential. Accordingly we replace the constants by the corresponding momentum dependent amplitudes satisfying (elastic) unitarity,

$$-a_i \rightarrow f_i^{(1)}(k) = \left(-\frac{1}{a_i} - ik \right)^{-1}, \quad i = 1, 3. \quad (27)$$

Here k is the centre-of-mass momentum, to be distinguished from p , which is the momentum of the in-medium nucleons in the rest frame of the nucleon under consideration, the two being related by $k^2 = m(\sqrt{m^2 + p^2} - m)/2$.

An even better approximation of the scattering amplitude is the effective range approximation that would result from including derivative terms in the effective theory [10, 11]. The corresponding replacement would read

$$-a_i \rightarrow f_i^{(2)}(k) = \left(-\frac{1}{a_i} + \frac{1}{2} r_i k^2 - ik \right)^{-1}, \quad i = 1, 3, \quad (28)$$

where $r_{1,3}$ are the effective ranges in the spin-singlet and -triplet s -waves. Without trying to relate the effective ranges to the coupling constants of the higher order terms in the effective Lagrangian, we shall take their values as well as those of the scattering lengths from experiment.

We thus get both the real and the imaginary parts of the pole position:

$$m^* - \frac{i}{2}\gamma = m - \frac{6\pi}{m} \int_0^{p_F} \frac{d^3p}{(2\pi)^3} \{ f_1(k) + f_3(k) \}, \quad (29)$$

where $f_i(k)$ stands for $f_i^{(1)}(k)$ or $f_i^{(2)}(k)$, according to our choice of the replacement (27) or (28). The width γ is to be interpreted as the damping rate of excitations with the quantum numbers of the nucleon. The numerical evaluation of (29) is shown in Figs. 3 and 4. In normal nuclear matter, we get $\Delta m \equiv m^* - m = -33 \text{ MeV}$ and $\gamma/2 = 110 \text{ MeV}$, with amplitudes in the effective range approximation (solid curves in Figs. 3 and 4).

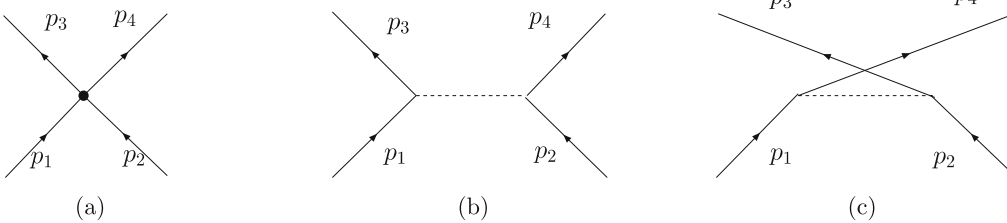


Fig. 2. Nucleon-nucleon scattering amplitude in the tree approximation

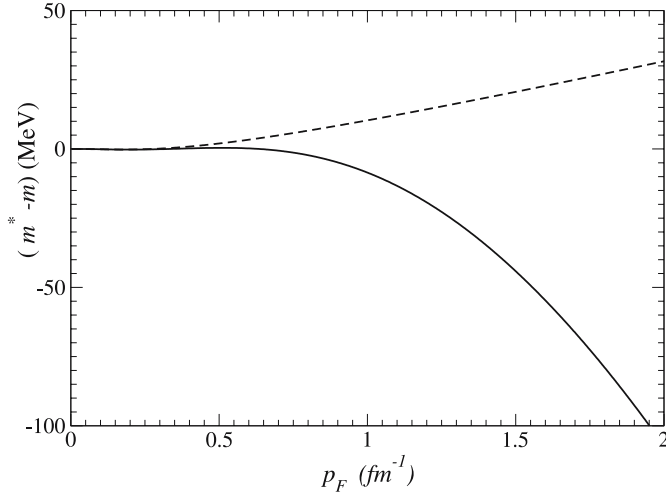


Fig. 3. Nucleon mass shift in nuclear matter as a function of Fermi momentum. The *dashed* and the *solid* curves are drawn with amplitudes $f_i^{(1)}(k)$ and $f_i^{(2)}(k)$ respectively

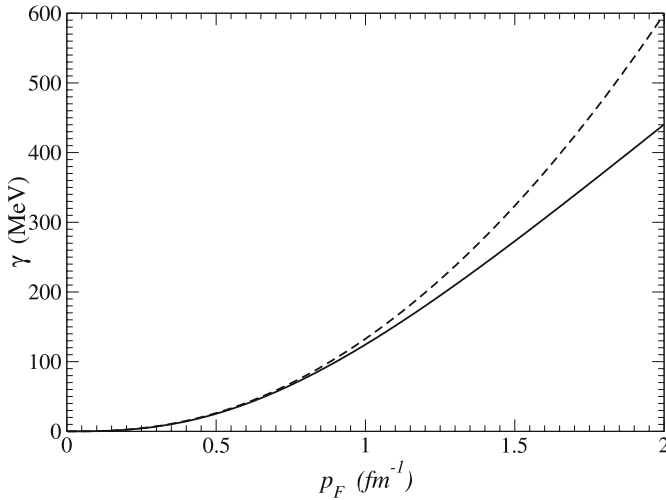


Fig. 4. Nucleon width in nuclear matter as a function of Fermi momentum. The *dashed* and the *solid* curves are drawn with the same amplitudes as in Fig. 3

Our results may be compared with those in the literature. There have been a number of calculations of the nucleon spectral function in terms of the (off-shell) NN scattering amplitude evaluated with the available NN potentials. We pick out from these works the real and the imaginary parts of the on-shell nucleon self-energy in normal nuclear matter at zero 3-momentum. Thus, Baldo et al. [19] give $\Delta m = -55$ MeV, $\gamma/2 = 63$ MeV (as reproduced in [22]). Benhar et al. [20] give $\Delta m = -68$ MeV, $\gamma/2 = 16$ MeV. Jong et al. [21] give $\Delta m = -65$ MeV, $\gamma/2 = 25$ MeV. Also, a phenomenological determination of the width in terms of the differential cross-section for NN scattering gives $\gamma/2 > 25$ MeV [22]. Finally, we compare with a calculation by us [23] based on the virial formula for the nucleon pole position, which gives $\Delta m = -37$ MeV, $\gamma/2 = 112$ MeV.

5 Discussion

The spectral function of a nucleon in nuclear matter depends on the interaction in a two-nucleon system. To treat this interaction in the framework of effective chiral field theory [7, 8], one has to derive first the effective potential from the effective Lagrangian, and then solve the dynamical equation with this potential. In this way one can restore the otherwise invalid power counting rule and accommodate at the same time the singularities of the scattering amplitude in the low energy region.

The failure of the earlier calculation of the effective nucleon mass [9] may now be understood in this framework as due to the representation of the two-nucleon scattering amplitude by the (leading term of the) effective potential itself. In this work we proceed to carry out the next dynamical step, replacing the potential by the unitarised scattering amplitude. By including the effective range terms in the s -wave amplitudes, we actually include the next-to-leading order terms also in the effective potential.

It will be noted that our mass-shift and width formulae do not involve any Pauli blocking effect, our improved scattering amplitude still being in *vacuum* and not in *medium*. An estimate of this effect can be made from the work of [24], where one finds a change of about 20% in the value of the potential energy per nucleon in nuclear matter, when the Pauli projection operators are withheld from the calculation. Incidentally, the phenomenological formula for the width in [22] involves the differential cross-section in *vacuum*, the Pauli blocking factors appearing only for the final nucleons. Such factors, however, do not appear in our formula, as it depends on the forward scattering amplitude, where the final particles occupy the states vacated by the initial ones.

The formula (29) bears a close resemblance to the one obtained from the virial expansion of the self-energy of the nucleon to first order [25]. In the virial formula, it is the full (spin averaged) forward amplitude that appears in the integral, which was evaluated in [23] using the phase shift analysis of the experimental data on NN scattering of [26], while the integral here includes only the part of this amplitude corresponding to s -waves in the effective range approximation. But in the low energy region over which the integrals are evaluated, this approximation for the s -waves agrees well with the phase shift analysis. Further, the higher partial waves contribute negligibly to the virial formula. We thus expect close agreement of the present results with those from the virial formula, as stated at the end of the previous section.

Appendix A

Here we describe the method to evaluate the density dependent part of the amplitude for the one-loop graphs. In this method we calculate the imaginary part of the Feynman amplitude and construct its real part by a dispersion

integral over it. Since we exclude the vacuum contribution and consider only the density dependent part, no subtraction is necessary for these integrals. The same result can be obtained in a simpler way by retaining only the density dependent part of the nucleon propagator in the amplitude and using the mass-shell delta-function in it to integrate out the energy component of the 4-momentum. But the dispersion method is more transparent in that it shows the cut structures where the contributions come from.

We now evaluate graph (e) of Fig. 1 in some detail, it being the prototype of all other graphs. It has the amplitude

$$\Pi(q)_{(e)} = -\frac{3\lambda^2}{4F_\pi^2} \Gamma(q), \quad (\text{A.1})$$

where

$$\Gamma(q)_{11} = i \int d^4x e^{iqx} \frac{1}{i} D(x)_{11} \gamma_5 \frac{1}{i} S(x)_{11} \gamma_5. \quad (\text{A.2})$$

Due to the presence of $\theta(\pm p_0)$ in (12) for $S(p)_{11}$, it is not convenient to transform the amplitude in momentum space. Instead, we integrate out the p_0 variable in $S(x)_{11}$ itself, getting

$$\begin{aligned} \frac{1}{i} S(x)_{11} &= \int \frac{d^3p}{(2\pi)^3 2\omega_p} \\ &\times [\{(\not{p} + m)(1 - n^-)e^{-ipx} + (\not{p} - m)n^+e^{ipx}\}\theta(x_0) \\ &- \{(\not{p} + m)n^-e^{-ipx} + (\not{p} - m)(1 - n^+)e^{ipx}\}\theta(-x_0)], \\ p_0 &\equiv \omega_p. \end{aligned} \quad (\text{A.3})$$

The analogous expression for the pion propagator is

$$\begin{aligned} \frac{1}{i} D(x)_{11} &= \int \frac{d^3k}{(2\pi)^3 2\omega_k} [\{(1 + n)e^{-ikx} + ne^{ikx}\}\theta(x_0) \\ &+ \{ne^{-ikx} + (1 + n)e^{ikx}\}\theta(-x_0)], \\ k_0 &\equiv \omega_k \end{aligned} \quad (\text{A.4})$$

where n is the pion distribution function, $n = (e^{\beta\omega_k} - 1)^{-1}$, $\omega_k = \sqrt{m_\pi^2 + k^2}$. Though $n^+(\omega_p)$ and $n(\omega_k)$ are zero for the medium we are interested in, we retain them at this stage for generality and symmetry.

With these expressions for the propagators, we may carry out both the x^0 and \mathbf{x} integrations in (A.2) giving the energy denominator and the 3-momentum delta-function, respectively. The imaginary part may now be read off and put in the form

$$\text{Im } \Gamma(q)_{11} = \pi \tanh(\beta(q_0 - \mu)/2) \text{Im } \Gamma(q), \quad (\text{A.5})$$

where

$$\begin{aligned} \text{Im } \Gamma(q) &= - \int \frac{d^3p}{(2\pi)^3 2\omega_p} \int \frac{d^3k}{(2\pi)^3 2\omega_k} (\not{p} - m) \\ &\times \{ (1 - n^- + n)\delta^{(4)}(q - p - k) \\ &+ (n^- + n)\delta^{(4)}(q - p + k) \}. \end{aligned} \quad (\text{A.6})$$

Here the first term corresponds to $\eta \rightarrow \pi N$ and the second one to $\eta + \pi \rightarrow N$, giving rise respectively to the discontinuity across the unitary and the ‘short’ cuts [27]. With $q_0 = E$ and for $\mathbf{q} = 0$, at which we work, these cuts extend in the E -plane over $E \geq m + m_\pi$ and $0 \leq E \leq m - m_\pi$. Note the opposite sign before n^- in the two terms.

The complete result for $\text{Im } \Gamma$ has also another piece, given by

$$\begin{aligned} &- \int \frac{d^3p}{(2\pi)^3 2\omega_p} \int \frac{d^3k}{(2\pi)^3 2\omega_k} (\not{p} + m) \\ &\times \{ (1 - n^+ + n)\delta^{(4)}(q + p + k) + (n^+ + n)\delta^{(4)}(q + p - k) \}, \end{aligned}$$

which is non-zero for $E < 0$ only and has no term proportional to n^- . Clearly this observation does not depend on the structure of the vertices in the loop diagrams. We thus have the general result that there is no spectral function to one loop for $E < 0$ in a medium with nucleons only.

The 3-momentum integrations in (A.6) can be carried out immediately so that we get

$$\begin{aligned} \text{Im } \Gamma(E) &= \pm \frac{f(E)}{E}, \\ f(E) &= \frac{\sqrt{\omega^2 - m^2}}{8\pi^2} (\gamma_0 \omega - m) n^-(\omega), \end{aligned} \quad (\text{A.7})$$

where $\omega = (E^2 + m^2 - m_\pi^2)/2E$ is the energy of the nucleon, ω_p , as restricted by the delta-functions. The \pm signs correspond to the unitary and the short cuts, respectively. Inserting this result in (11), we get the spectral representation for $\Gamma(E)$,

$$\Gamma(E) = \int_{m+m_\pi}^{\infty} \frac{dE' f(E')}{E'(E' - E)} - \int_0^{m-m_\pi} \frac{dE' f(E')}{E'(E' - E)}. \quad (\text{A.8})$$

The range of the second integral can be mapped onto that of the first one by the inverse transformation $E' \rightarrow (m^2 - m_\pi^2)/E'$. Noting that ω and hence $f(E)$ is form invariant under this transformation, we have

$$\begin{aligned} \Gamma(E) &= \int_{m+m_\pi}^{\infty} \frac{dE'}{E'} f(E') \\ &\times \left(\frac{1}{E' - E} - \frac{1}{(m^2 - m_\pi^2)/E' - E} \right). \end{aligned} \quad (\text{A.9})$$

Once we are on the unitary cut, the kinematics is determined by the first δ -function in (A.6). For $\mathbf{q} = 0$, it gives

$$E' = \sqrt{m^2 + p^2} + \sqrt{m_\pi^2 + p^2}, \quad (\text{A.10})$$

so that E' and p are the total energy and the 3-momentum in the centre-of-mass frame of the intermediate πN system. The distribution function restricts the upper limit of the integral to the energy given by (A.10) with p replaced by the Fermi momentum p_F . Now setting $m_\pi = 0$, (A.9) gives Γ to leading order as

$$\Gamma(E) = -\frac{1}{4\pi^2} \int_0^{p_F} \frac{dp p^2}{(E-m)^2 - p^2} (1 - \gamma_0), \quad (\text{A.11})$$

which is finite at $E = m$.

Appendix B

The result (24) corresponds to s -wave amplitudes. To get the exact combination of these amplitudes, we recall the partial wave analysis of \overline{M} . Following the usual convention, we write it in the centre-of-mass system as

$$\overline{M} = 8\pi W \left(\frac{3}{2} \bar{f}^{(I=1)}(k) + \frac{1}{2} \bar{f}^{(I=0)}(k) \right), \quad (\text{B.1})$$

where W and k are the total energy and momentum in the c.m. frame. Each of the isospin amplitudes above has the partial wave expansion [28]

$$\bar{f}^{(I)}(k) = 2 \cdot \frac{1}{4} \sum_{j,s,l} (2j+1) f_{l,l}^{Ijs}(k). \quad (\text{B.2})$$

Here the factor 2 is due to the identity of the scattering particles. The total angular momentum j is obtained by coupling the total spin and orbital angular momenta s and l . In general, f^{Ijs} is a 2×2 matrix in l space, whose diagonal elements enter the sum in (26). The quantum numbers of the partial wave amplitudes are governed by the antisymmetry of the total wave function,

$$(-1)^l \cdot (-1)^{1-s} \cdot (-1)^{1-I} = -1.$$

Thus, in the s -wave approximation, (25) reduces to

$$\overline{M} = 6\pi W \{f_1(k) + f_3(k)\} \xrightarrow{\text{threshold}} -12\pi m(a_1 + a_3), \quad (\text{B.3})$$

where $f_{1,3}$ are the spin-singlet and -triplet s -wave amplitudes and $a_{1,3}$ the corresponding scattering lengths. Experimentally $a_1 = -23.74$ fm and $a_3 = 5.31$ fm [29]. We also need the experimental values for the effective ranges, $r_1 = 2.7$ fm and $r_3 = 1.70$ fm [29].

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References

1. S. Weinberg, *Physica A* **96**, 327 (1979)
2. J. Gasser, H. Leutwyler, *Ann. Phys.* **158**, 142 (1984)
3. J. Gasser, H. Leutwyler, *Nucl. Phys. B* **250**, 465 (1985)
4. T. Becher, H. Leutwyler, *JHEP* **0106**, 17 (2001) [[hep-ph/0103263](#)]
5. E. Epelbaum, *Prog. Part. Nucl. Phys.* **57**, 654 (2006) [[nucl-th/0509032](#)]
6. G. Ecker, *Effective Field Theories and Chiral Perturbation Theory*, lectures given at Troisième Cycle de la Physique, 1998
7. S. Weinberg, *Phys. Lett. B* **251**, 288 (1990)
8. S. Weinberg, *Nucl. Phys. B* **363**, 3 (1991)
9. D. Montano, H.D. Politzer, M.B. Wise, *Nucl. Phys. B* **375**, 507 (1992)
10. D.B. Kaplan, M.J. Savage, M.B. Wise, *Nucl. Phys. B* **478**, 629 (1996)
11. D.B. Kaplan, M.J. Savage, M.B. Wise, *Nucl. Phys. B* **534**, 329 (1998)
12. N.P. Landsmann, C.G. van Weert, *Phys. Rep.* **145**, 141 (1987)
13. B.L. Ioffe, *Nucl. Phys. B* **188**, 317 (1981)
14. Y. Chung, H.G. Dosch, M. Kremer, D. Schall, *Nucl. Phys. B* **197**, 55 (1982)
15. G. Ecker, J. Gasser, A. Pich, E. de Rafael, *Nucl. Phys. B* **321**, 311 (1989)
16. G. Ecker, J. Gasser, H. Leutwyler, A. Pich, E. de Rafael, *Phys. Lett. B* **223**, 425 (1989)
17. R.L. Kobes, G.W. Semenoff, N. Weiss, *Z. Phys. C* **29**, 371 (1985)
18. A.L. Fetter, J.D. Walecka, *Quantum Theory of Many-particle Systems* (McGraw-Hill, New York, 1971)
19. M. Baldo, I. Bombaci, G. Giansiracusa, U. Lombardo, C. Mahoux, R. Sartor, *Nucl. Phys. A* **545**, 741 (1992)
20. O. Benhar, A. Fabrocini, S. Fantoni, *Nucl. Phys. A* **550**, 201 (1992)
21. F. de Jong, H. Lenske, *Phys. Rev. C* **56**, 154 (1997)
22. J. Lehr, U. Mosel, *Phys. Rev. C* **69**, 024603 (2004)
23. S. Mallik, A. Nyffeler, M.C.M. Rentmeester, S. Sarkar, *Eur. Phys. J. A* **22**, 371 (2004)
24. R. Machleidt, *Advances in Nuclear Physics* (Plenum Press, New York, 1989), vol. 19, p. 189
25. S. Mallik, *Eur. Phys. J. C* **24**, 143 (2002)
26. V.G.J. Stoks, R.A.M. Klomp, M.C.M. Rentmeester, J.J. de Swart, *Phys. Rev. C* **48**, 792 (1993)
27. H.A. Weldon, *Phys. Rev. D* **28**, 2007 (1983)
28. S. Weinberg, *The Quantum Theory of Fields* (Cambridge University Press, Cambridge, 1995), vol. I, p. 151
29. A. Bohr, B.R. Mottelson, *Nuclear Structure* (W.A. Benjamin, Inc., New York, 1969), vol. I